**Properties of Parallelograms (pp 330-333)**

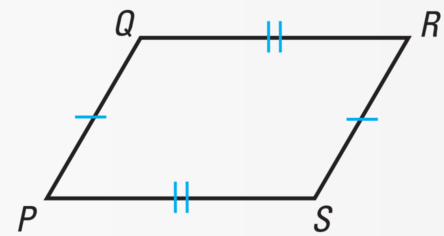
- I can define a parallelogram.
- I can state and apply the properties of a parallelogram.

## THEOREMS ABOUT PARALLELOGRAMS

**THEOREM 6.2**

If a quadrilateral is a parallelogram, then its **opposite sides** are congruent.

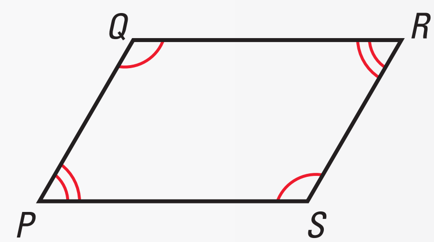
$\overline{PQ} \cong \overline{RS}$ and $\overline{SP} \cong \overline{QR}$




**THEOREM 6.3**

If a quadrilateral is a parallelogram, then its **opposite angles** are congruent.

$\angle P \cong \angle R$ and $\angle Q \cong \angle S$

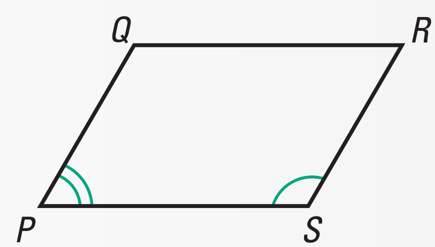



**THEOREM 6.4**

If a quadrilateral is a parallelogram, then its **consecutive angles** are supplementary.

$m\angle P + m\angle Q = 180°$, $m\angle Q + m\angle R = 180°$,

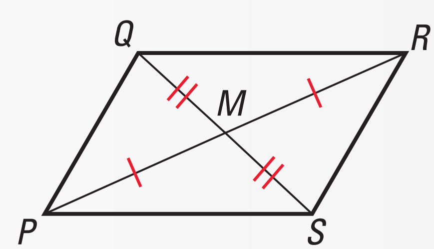
$m\angle R + m\angle S = 180°$, $m\angle S + m\angle P = 180°$




**THEOREM 6.5**

If a quadrilateral is a parallelogram, then its diagonals bisect each other.

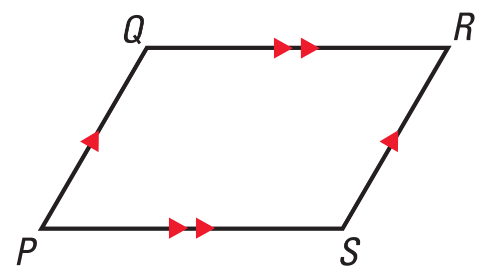
$\overline{QM} \cong \overline{SM}$ and $\overline{PM} \cong \overline{RM}$




A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.

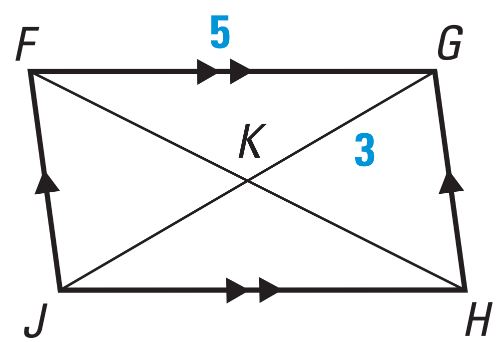
**Properties of Parallelograms (pp 330-333)**

When you mark diagrams of quadrilaterals, use matching arrowheads to indicate which sides are parallel. For example, in the diagram at the right, $\overline{PQ} \parallel \overline{RS}$ and $\overline{QR} \parallel \overline{SP}$. The symbol ▱*PQRS* is read "parallelogram *PQRS*."




## EXAMPLE 1 Using Properties of Parallelograms

*FGHJ* is a parallelogram. Find the unknown length. Explain your reasoning.




**a.** *JH*

**b.** *JK*

**SOLUTION**

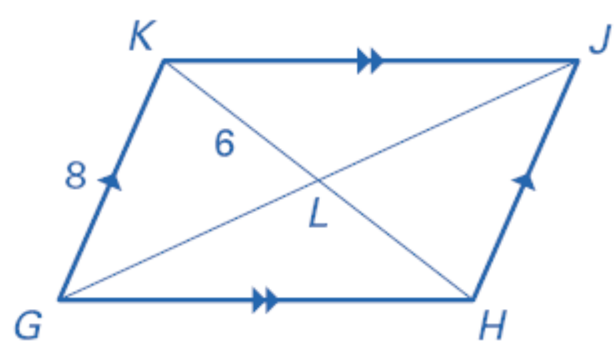
**a.** $JH = FG$     **Opposite sides of a ▱ are ≅.**

$JH = 5$     **Substitute 5 for *FG*.**

**b.** $JK = GK$     **Diagonals of a ▱ bisect each other.**

$JK = 3$     **Substitute 3 for *GK*.**

**1.** *GHJK* is a parallelogram. Find the JH & LH.



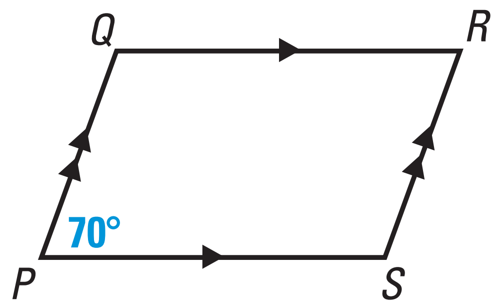
Properties of Parallelograms (pp 330-333)

## EXAMPLE 2 *Using Properties of Parallelograms*

*PQRS* is a parallelogram.
Find the angle measure.

**a.** $m\angle R$

**b.** $m\angle Q$




### SOLUTION

**a.** $m\angle R = m\angle P$ — **Opposite angles of a ▱ are ≅.**

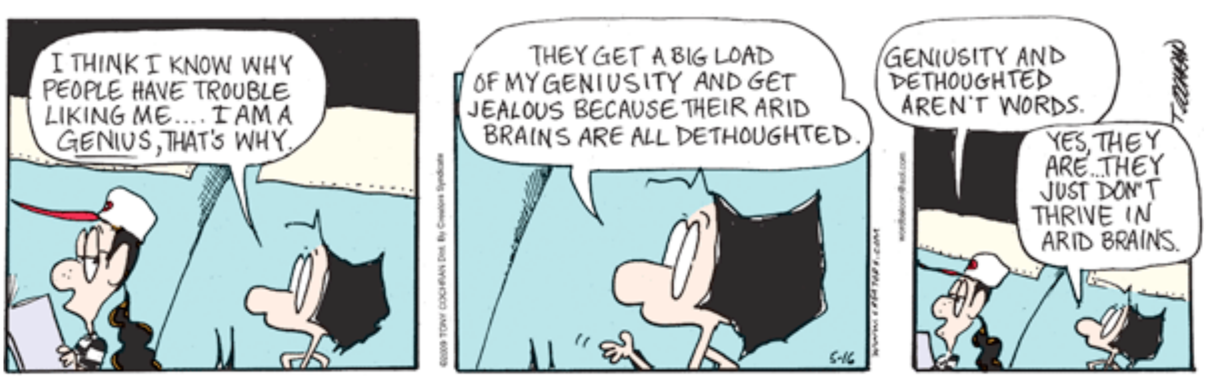
$m\angle R = 70°$ — **Substitute 70° for $m\angle P$.**

**b.** $m\angle Q + m\angle P = 180°$ — **Consecutive ∠s of a ▱ are supplementary.**

$m\angle Q + 70° = 180°$ — **Substitute 70° for $m\angle P$.**

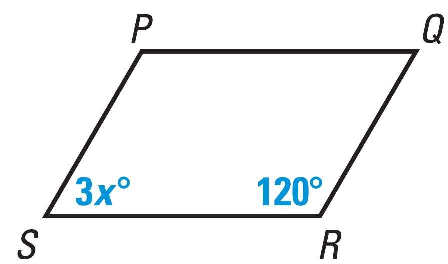
$m\angle Q = 110°$ — **Subtract 70° from each side.**

**2.** In ▱$ABCD$, $m\measuredangle C = 105°$. Find $m\measuredangle A$ & $m\measuredangle D$.



## EXAMPLE 3 *Using Algebra with Parallelograms*

*PQRS* is a parallelogram.
Find the value of *x*.




### SOLUTION

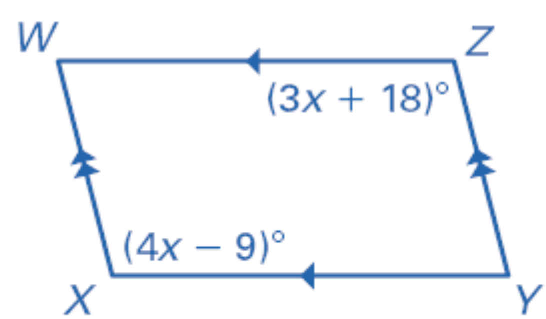
$m\angle S + m\angle R = 180°$ **Consecutive angles of a ▱ are supplementary.**

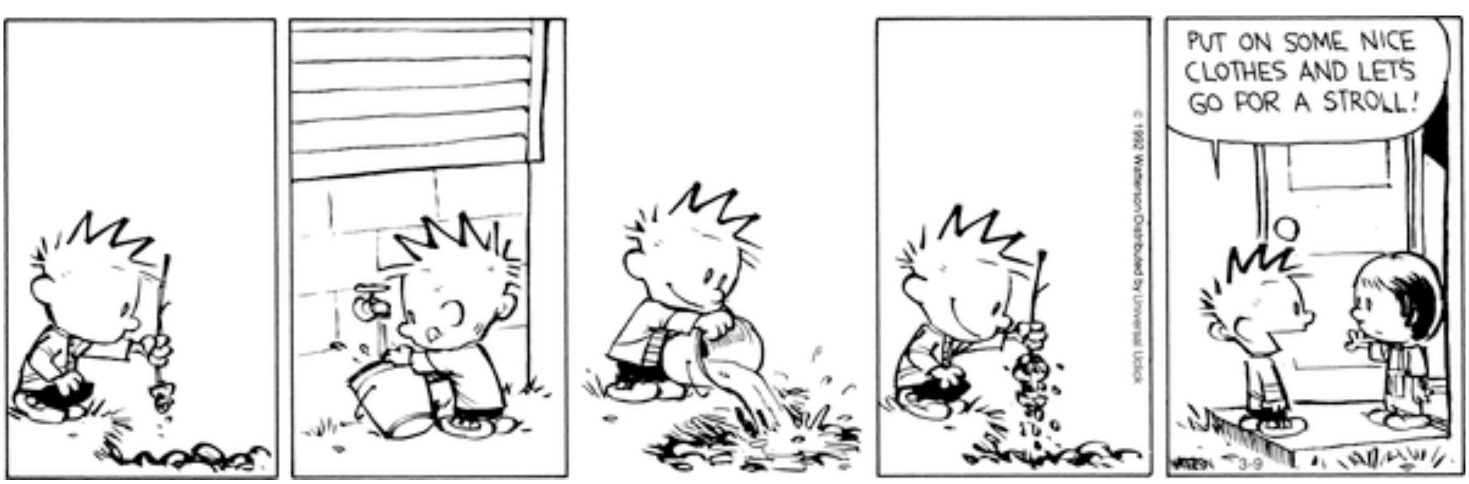
$3x + 120 = 180$ **Substitute 3*x* for *m*∠*S* and 120 for *m*∠*R*.**

$3x = 60$ **Subtract 120 from each side.**

$x = 20$ **Divide each side by 3.**

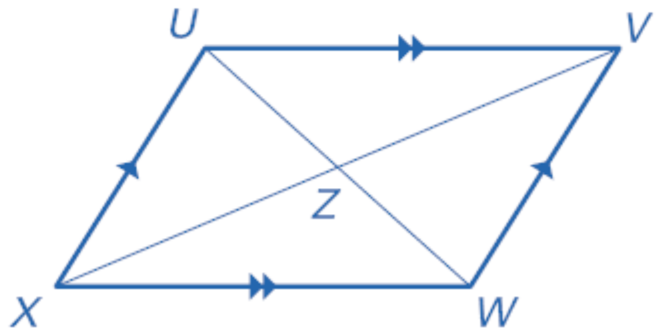
**3.** WXYZ is a parallelogram. Find the value of x.



# Properties of Parallelograms (pp 330-333)

UVWX is a parallelogram.




**4.** If XU = 15 and UW = 28, find WZ.

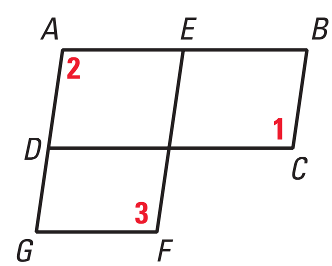
**5.** If $m\measuredangle VWX = 120°$, find $m\measuredangle WXU$.

**6.** If $m\measuredangle UVW = 55°$ & $m\measuredangle VWX = 7x - 8$, find x.

## EXAMPLE 4 *Proving Facts about Parallelograms*

**GIVEN ▶** *ABCD* and *AEFG* are parallelograms.

**PROVE ▶** $\angle 1 \cong \angle 3$




***Plan*** Show that both angles are congruent to $\angle 2$. Then use the Transitive Property of Congruence.

**SOLUTION**

***Method 1*** Write a two-column proof.

| Statements | Reasons |
|---|---|
| **1.** *ABCD* is a ▱. *AEFG* is a ▱. | **1.** Given |
| **2.** $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 3$ | **2.** Opposite angles of a ▱ are ≅. |
| **3.** $\angle 1 \cong \angle 3$ | **3.** Transitive Property of Congruence |

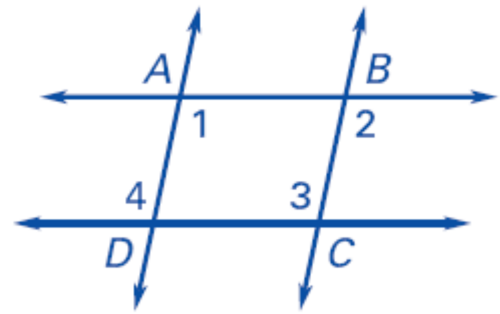
***Method 2*** Write a paragraph proof.

*ABCD* is a parallelogram, so $\angle 1 \cong \angle 2$ because opposite angles of a parallelogram are congruent. *AEFG* is a parallelogram, so $\angle 2 \cong \angle 3$. By the Transitive Property of Congruence, $\angle 1 \cong \angle 3$.

**Properties of Parallelograms (pp 330-333)**

**7.** Given: ABCD is a ▱.

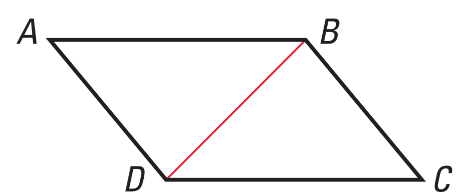
Prove: $\measuredangle 2 \cong \measuredangle 4$




**EXAMPLE 5** ***Proving Theorem 6.2***

**GIVEN ▶** *ABCD* is a parallelogram.

**PROVE ▶** $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$




**SOLUTION**

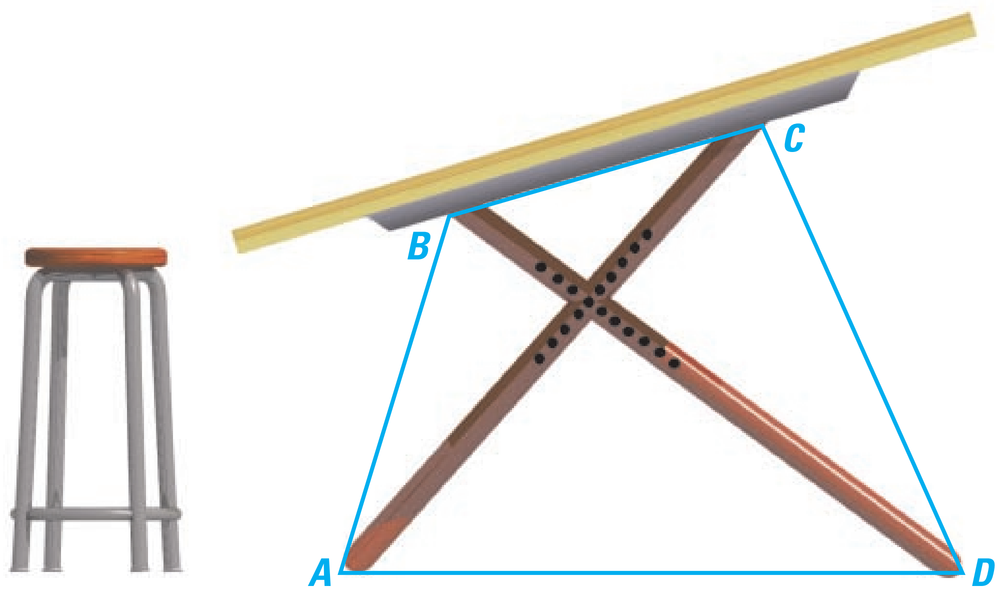
| Statements | Reasons |
|---|---|
| **1.** *ABCD* is a ▱. | **1.** Given |
| **2.** Draw $\overline{BD}$. | **2.** Through any two points there exists exactly one line. |
| **3.** $\overline{AB} \parallel \overline{CD}$, $\overline{AD} \parallel \overline{CB}$ | **3.** Definition of parallelogram |
| **4.** $\angle ABD \cong \angle CDB$, $\angle ADB \cong \angle CBD$ | **4.** Alternate Interior Angles Theorem |
| **5.** $\overline{DB} \cong \overline{DB}$ | **5.** Reflexive Property of Congruence |
| **6.** $\triangle ADB \cong \triangle CBD$ | **6.** ASA Congruence Postulate |
| **7.** $\overline{AB} \cong \overline{CD}$, $\overline{AD} \cong \overline{CB}$ | **7.** Corresponding parts of ≅ ⚠ are ≅. |

**Properties of Parallelograms (pp 330-333)**

**9.** Sketch parallelogram ABCD with rays $\overrightarrow{BAE}$ & $\overrightarrow{DCF}$. You can use linear pairs to show that $m\measuredangle EAD + m\measuredangle DAB = m\measuredangle FCB + m\measuredangle DCB$. What postulate or theorem can you then use with the substitution and subtraction properties of equality and definition of congruence to show that $\measuredangle EAD \cong \measuredangle FCB$?

## EXAMPLE 6 *Using Parallelograms in Real Life*

**FURNITURE DESIGN** A drafting table is made so that the legs can be joined in different ways to change the slope of the drawing surface. In the arrangement below, the legs $\overline{AC}$ and $\overline{BD}$ do *not* bisect each other. Is $ABCD$ a parallelogram?



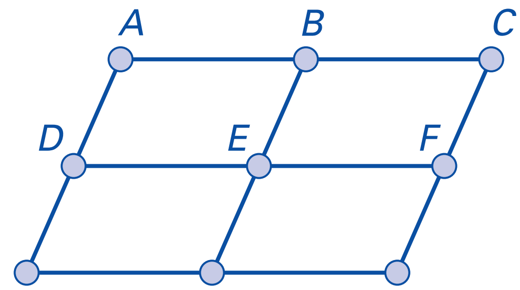

**SOLUTION**

No. If $ABCD$ were a parallelogram, then by Theorem 6.5 $\overline{AC}$ would bisect $\overline{BD}$ and $\overline{BD}$ would bisect $\overline{AC}$.

**Properties of Parallelograms (pp 330-333)**

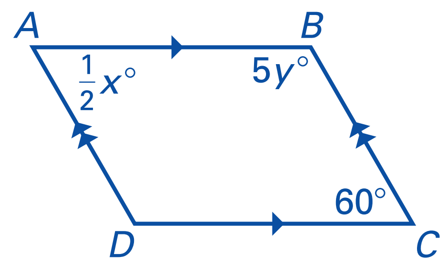
**10.** A four-sided concrete slab has consecutive angle measures of $85°, 94° \& 85°$. Is the slab a parallelogram? Explain.

**11.** Each circle in the crystal lattice shown represents a molecule. *ABED* and *BCFE* are parallelograms. *A*, *B*, and *C* are collinear, as are *D*, *E*, and *F*. Must *ACFD* be a parallelogram? Explain.




**13.** _____ What are the values of *x* and *y* in parallelogram *ABCD*?

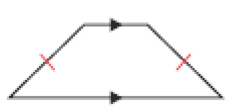
**A.** x = 30, y = 24
**B.** x = 60, y = 24
**C.** x = 120, y = 24
**D.** x = 120, y = 12
**E.** x = 240, y = 12




**12.** Write the definition of a parallelogram.

**Decide whether the figure is a parallelogram. If it is not, explain why not.**

**13.**



**14.**